Stochastic temporary stabilization: Undiversifiable devaluation and income risks

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Abstract

This paper presents a stochastic model of exchange-rate-based inflation stabilization that explicitly recognizes the role of uncertainty in the dynamics of both the exchange rate and labor income. In our setting, we assume that the exchange rate is driven by a mixed diffusion-jump process, and the consumer’s labor income follows a geometric Brownian motion. We suppose that contingent-claims markets for hedging against inflation and future income are not available, so financial markets are incomplete. We examine consumption and portfolio shares dynamics when a stabilization plan is implemented and labor income is uncertain. We also assess the effects on welfare of exogenous shocks in expectations of both devaluation and income. Finally, we use the proposed model to carry out a Monte Carlo simulation experiment that explains the observed orders of magnitude of consumption booms, under uncertain labor income, for the Mexican case between 1989 and 1994.

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1. Introduction

In this paper, we develop a stochastic model of exchange-rate-based inflation stabilization emphasizing the role of uncertainty in the dynamics of both the exchange rate and the
consumer’s labor income. In our setting, the exchange rate is driven by a mixed diffusion-jump process, and future labor income is governed by a geometric Brownian motion. By considering the whole distribution of the exchange rate and the consumer’s labor income, we might even examine those events that, in spite of their small probability of occurrence, could lead to significant impacts on temporary stabilization outcomes.

We suppose that financial derivatives for hedging against inflation and future income are unavailable, so markets are incomplete. The model also assumes that revenue raised by seignorage is wasted in unproductive government purchases. Using a framework of risk adverse agents, we examine the equilibrium dynamics of consumption and wealth when a stabilization program is implemented. We also study the effects on consumption and economic welfare of once-and-for-all changes in the parameters that determine the expectations, namely, the mean expected rate of devaluation, the exchange-rate volatility, the probability of devaluation, the mean expected size of a possible devaluation, and future labor income.

Among the few studies considering a stochastic setting of exchange-rate-based stabilization, we have, for example, Drazen and Helpman (1988), Calvo and Drazen (1997), Mendoza and Uribe (1996, 1998), and Venegas-Martinez (2001). All of these models share important similarities: (1) markets of contingent-claims are unavailable, (2) the revenue raised by seignorage is not rebated back to the agents, and (3) policy variables are stochastic.

The inflation stabilization programs, which took place in Argentina, Brazil, Chile, Uruguay, Israel, and Mexico between the 1970s and the 1990s, have been widely documented. There is a large literature reporting key empirical regularities associated with these programs: see e.g., Helpman and Razin (1987), Kiguel and Liviatan (1992), and Végh (1992). There is also an increasing number of models providing explanations for such empirical regularities: see e.g., Calvo (1986), Calvo and Végh (1993, 1994), Reinhart and Végh (1993, 1996), Rodriguez (1982), Roldos (1995), Uribe (1997), Lahiri (2001), Rebelo and Végh (1995), Matsuyama (1991), and De Gregorio et al. (1998).

Our modeling has several distinctive features in studying the effects of uncertainty in exchange-rate-based inflation stabilization programs: (1) it takes into account all risk factors in the exchange rate and labor income, providing a more realistic stochastic environment; (2) it derives tractable closed-form solutions, making easier the understanding of the key issues in the analysis of temporary stabilization; (3) it examines the effects on temporary stabilization plans of uncertain labor income; and (4) it explains the observed orders of magnitude of consumption booms by using Monte Carlo simulation methods.

The paper is organized as follows. In the next section, we work out a Ramsey-type, one-good, cash-in-advance stochastic model where agents have expectations of devaluation driven by a mixed diffusion-jump process. We also supposed that future labor income evolves in accordance with a geometric Brownian motion. In Section 3, we solve the consumer’s choice problem. In Section 4, we undertake policy experiments. In Section 5, we examine the welfare implications. In Section 6, we study the dynamic behavior of wealth and consumption addressing a number of exchange-rate policy issues. In Section 7, we carry out a
Monte Carlo simulation of consumption behavior when labor income is paid at a stochastic rate. In Section 8, we draw conclusions, acknowledge limitations, and make suggestions for further research. Two appendices contain some technical details on the consumer’s choice problem.

2. The main assumptions of the model

In order to derive analytically tractable solutions in a stochastic Ramsey-type model, the structure of the price-taking economy will be kept as simple as possible. The main assumptions of the model are stated in such a way that the key issues of temporary stabilization under uncertain income are easier to understand.

2.1. Price level dynamics

Let us consider a small open economy with a single infinite-lived household in a world with a single perishable consumption good. We assume that the good is freely traded, and its domestic price level, \( P_t \), is determined by the purchasing power parity condition, namely, \( P_t = P_t^* e_t \), where \( P_t^* \) is the foreign-currency price of the good in the rest of world, and \( e_t \) is the nominal exchange rate. We will assume, for the sake of simplicity, that \( P_t^* \) is equal to 1. We also assume that the initial value of the exchange rate, \( e_0 \), is known and equal to 1.

We suppose that the number of expected devaluations, i.e., jumps in the exchange rate, per unit of time, follows a Poisson process \( N_t \) with intensity \( \lambda \), so

\[
\operatorname{IP}^{(N)}\{\text{one unit jump during } dt\} = \operatorname{IP}^{(N)}\{dN_t = 1\} = \lambda dt + o(dt),
\]

whereas

\[
\operatorname{IP}^{(N)}\{\text{no jump during } dt\} = \operatorname{IP}^{(N)}\{dN_t = 0\} = 1 - \lambda dt + o(dt).
\]

Thus, \( \mathbb{E}^{(N)}[dN_t] = \text{Var}^{(N)}[dN_t] = \lambda dt \). We set the initial number of jumps identically equal to zero, that is, \( N_0 = 0 \).

We assume that the consumer perceives that the expected inflation rate, \( dP_t/P_t \), and consequently, the expected rate of devaluation, \( de_t/e_t \), follows a geometric Brownian motion with Poisson jumps. Let us consider a Wiener process \( (Z_t)_{t \geq 0} \) defined on some fixed filtered probability space \( (\Omega^{(Z)}, \mathcal{F}^{(Z)}, (\mathcal{F}^{(Z)}_t)_{t \geq 0}, \text{IP}^{(Z)}) \), and assume that inflation is driven by

\[
\frac{dP_t}{P_t} = \frac{de_t}{e_t} = \pi dt + \sigma_P dZ_t + \gamma dN_t,
\]

where \( \pi \) is the mean expected rate of devaluation conditional on no jumps, \( \sigma_P \) is the instantaneous volatility of the expected price level, and \( \gamma \) is the mean expected size of a exchange-rate jump. Process \( Z_t \) is supposed to be independent of \( N_t \). In what follows, \( \pi \), \( \sigma_P \), \( \lambda \) and \( \gamma \) are all supposed to be positive constants.
2.2. Real monetary balances

The agent holds real cash balances, \( m_t = M_t / P_t \), where \( M_t \) is the nominal stock of money. The stochastic rate of return of holding real cash balances, \( dR_m \), is given by the percentage change in the price of money in terms of goods. By applying Itô’s lemma for diffusion-jump processes to the inverse of the price level, with (3) as the underlying process (see Appendix A, formula (A.2)), we obtain

\[
dR_m = \left( \frac{M_t}{P_t} \right) \left( -\pi + \sigma_p^2 \right) dt - \sigma_p dZ_t - \left( \frac{\gamma}{1 + \gamma} \right) dN_t. \tag{4}
\]

2.3. International bonds

The agent also holds an international bond, \( b_t \), that pays a risk-free real interest rate, \( r \), which is constant for all terms, satisfying

\[
db_t = rb_t dt, \quad b_0 > 0.
\tag{5}
\]

That is, the bond pays \( r \) units of the consumption good per unit of time. Of course, the agent takes \( r \) as given.

2.4. A cash-in-advance economy

Consider a cash-in-advance constraint of the Clower–Lucas–Feenstra form:

\[
m_t = \alpha c_t, \tag{6}
\]

where \( c_t \) is consumption, and \( \alpha > 0 \) is the time that money must be held to finance consumption. Condition (6) is critical in linking exchange-rate policy and consumption. In such a case, devaluation acts as a stochastic tax rate on real cash balances.

2.5. Uncertain labor income

For the sake of simplicity, we may assume that the representative consumer manages and works in his/her own business. We suppose labor income, \( y_t \), is transformed into real assets, \( a_t \), at an uncertain rate \( v_t \), in such a way that

\[
y_t = v_t a_t,
\]

and \( v_t \) is driven by a geometric Brownian motion. Let \( (U_t)_{t \geq 0} \) be a Wiener process defined on a fixed filtered probability space \( (\Omega^U, \mathcal{F}^U, (\mathcal{F}^U_t)_{t \geq 0}, \mathbb{P}^U) \), we assume that the rate of variability of income follows

\[
\frac{dv_t}{v_t} = \tilde{v} dt + \sigma_v d\tilde{Z}_t, \quad v_0 > 0, \tag{7}
\]

where

\[
\tilde{Z}_t = \rho Z_t + \sqrt{1 - \rho^2} U_t \tag{8}
\]
\[
\text{Cov}\left( dZ_t, d\left( \rho Z_t + \sqrt{1 - \rho^2} U_t \right) \right) = \rho dt,
\]

where \( \rho \in (-1, 1) \) is the correlation between changes in inflation and changes in labor income. Notice that an increase in the rate of devaluation will produce a higher depreciation in real cash balances. This, in turn, will reduce real assets, which could affect the income–assets ratio, given by \( \nu_t \). Processes \( N_t, Z_t, \) and \( U_t \) are supposed to be pairwise independent.

### 3. The consumer’s choice problem

In this section, we characterize the household’s optimal decisions on consumption and portfolio shares. We derive closed-form solutions to examine the dynamic implications of uncertainty. These explicit solutions make easier the understanding of the main issues of temporary stabilization programs.

#### 3.1. Intertemporal budget constraint

The stochastic consumer’s wealth accumulation in terms of the portfolio shares, \( w_t = \frac{m_t}{a_t}, 1 - w_t = \frac{b_t}{a_t} \), and consumption, \( c_t \), is given by the following system:

\[
\begin{align}
\begin{aligned}
d a_t &= a_t w_t dR_m + a_t (1 - w_t) dR_b + (\nu_t a_t - c_t) dt, \quad a_0 = m_0 + b_0 > 0, \\
d \nu_t &= \bar{\nu} \nu_t dt + \sigma \nu_t \left( \rho dZ_t + \sqrt{1 - \rho^2} dU_t \right), \quad \nu_0 > 0,
\end{aligned}
\end{align}
\]

where \( dR_b = db_t/b_t \). By substituting (4), (5) and (6) into the first equation of (10), we get

\[
d a_t = a_t \left[ (r - \zeta w_t + \nu_t) dt - w_t \sigma_p dZ_t - w_t \left( \frac{\gamma}{1 + \gamma} \right) dN_t \right],
\]

where

\[\zeta = \alpha^{-1} + r + \pi - \sigma^2.\]

#### 3.2. The felicity index

The von Neumann–Morgenstern utility at time \( t \), \( V_t \), of the competitive risk-averse consumer is assumed to have the time-separable form:

\[
V_t = E \left\{ \int_t^\infty \log(c_s) e^{-rs} ds \bigg| \mathcal{F}_t \right\},
\]

where \( \mathcal{F}_t = \mathcal{F}_t^Z \otimes \mathcal{F}_t^U \) stands for all available information at \( t \). We consider the logarithmic utility function in order to derive closed-form solutions and make the analysis easy to manage. The agent’s subjective discount rate has been set equal to the interest rate, \( r \), to avoid unnecessary technical difficulties.
3.3. The Hamilton–Jacobi–Bellman equation

In this case, the Hamilton–Jacobi–Bellman equation for the stochastic optimal control problem of maximizing the agent’s lifetime expected utility, (12), subject to the intertemporal budget constraint, (10), is

\[
\lambda I(a_t, v_t, t) - I_v(a_t, v_t, t) \tilde{v} v_t - \frac{1}{2} I_{vv}(a_t, v_t, t)v_t^2 \sigma_v^2 - I_a(a_t, v_t, t)a_t(r + v_t)
\]

\[
= \max_w \left\{ \log(x^{-1}a_tw_t)e^{-rt} - I_a(a_t, v_t, t)a_t\zeta w_t + \frac{1}{2} I_{aa}(a_t, v_t, t)a_t^2w_t^2\sigma_p^2 - I_{av}(a_t, v_t, t)a_tv_v\sigma_p\sigma_v\rho + \lambda I \left( a_t \left( \frac{1 + \gamma(1 - w_t)}{1 + \gamma} \right), v_t, t \right) \right\},
\]

(13)

where

\[
I(a_t, v_t, t) = \max_w \mathbb{E}_t \left\{ \int_t^\infty \log(x^{-1}a_sw_s)e^{-rs}ds \bigg| \mathcal{F}_t \right\}
\]

is the agent’s indirect utility function (or welfare function) and \(I_a(a_t, v_t, t)\) is the co-state variable.

3.4. Reduction in the dimension of the problem

Given the exponential time discounting in (14), we specify \(I(a_t, v_t, t)\) in a time-separable form as

\[
I(a_t, v_t, t) = F(a_t, v_t)e^{-rt}.
\]

Hence, (13) is transformed into

\[
(\lambda + r)F(a_t, v_t) - F_v(a_t, v_t)\tilde{v}v_t - \frac{1}{2} F_{vv}(a_t, v_t)v_t^2\sigma_v^2 = F_a(a_t, v_t)a_t(r + v_t)
\]

\[
= \max_w \left\{ \log(x^{-1}a_tw_t) - F_a(a_t, v_t)a_t\zeta w_t + \frac{1}{2} F_{aa}(a_t, v_t)a_t^2w_t^2\sigma_p^2
\]

\[
-F_{av}(a_t, v_t)a_tv_v\sigma_p\sigma_v\rho + \lambda F \left( a_t \left( \frac{1 + \gamma(1 - w_t)}{1 + \gamma} \right), v_t \right) \right\}.
\]

(15)

We postulate

\[
F(a_t, v_t) = \theta_0 + \theta_1 \log(a_t) + \phi(v_t; \theta_2, \theta_3),
\]

(16)

where \(\theta_0, \theta_1,\) and \(\phi(v_t; \theta_2, \theta_3)\) are to be determined from Eq. (15). Coefficients \(\theta_2\) and \(\theta_3\) must satisfy \(\phi(v_0) = 0\) and \(\phi'(v_0) = 0\). Substituting (16) into (15), we have

\[
r(\theta_0 + \theta_1 + \log(a_t)) - \theta_1 \tilde{v} + \frac{1}{2} \theta_1 \sigma_v^2 + \theta_1 r + r\phi(v_t) - \phi'(v_t)\tilde{v}v_t - \frac{1}{2} \phi''(v_t)v_t^2\sigma_v^2
\]

\[+ r\theta_1 \log(v_t) - \theta_1 v_t = \max_w \left\{ \log(x^{-1}a_tw_t) - \theta_1 \zeta w_t - \frac{1}{2} \theta_1 w_t^2\sigma_p^2
\]

\[+ \lambda \theta_1 \log \left( \frac{1 + \gamma(1 - w_t)}{1 + \gamma} \right) \right\}.
\]

(17)
3.5. First-order conditions and coefficients determination

After computing the first-order conditions, we find that \( w_t = w \) is time-invariant, and

\[
\frac{1}{\theta_1 w} - \frac{\lambda^2}{1 + \gamma(1 - w)} = \zeta + w\sigma^2_p. \tag{18}
\]

We choose now \( \phi(v_t) \) as a solution of

\[
 r\phi(v_t) - \phi'(v_t)\ddot{v}_t \left( 1 - \frac{2}{v} \sigma^2_v \right) + r\theta_1 \log(v_t) - \theta_1 v_t = 0. \tag{19}
\]

Coefficients \( \theta_0 \) and \( \theta_1 \) are determined from (15) after substituting optimal \( w^* \). Thus, \( \theta_1 = r^{-1} \), so the coefficient of \( \log(a_t) \) in (17) becomes zero, and

\[
\theta_0 = \frac{1}{r} \log(x^{-1}w^*) - \frac{1}{r^2} \left[ (x^{-1} + \pi - \sigma^2_p)w^* + \frac{1}{2} \left( w^* \sigma_p \right)^2 - \ddot{v} - r + \frac{1}{2} \sigma^2_v \right.
\]

\[
- \frac{\lambda}{\gamma} \log \left( \frac{1 + \gamma(1 - w^*)}{1 + \gamma} \right) \bigg]. \tag{20}
\]

Logarithmic utility implies that \( w \) depends only upon the parameters determining the stochastic characteristics of the economy, and hence, \( w \) is constant. In other words, the consumer’s attitude toward currency risk is independent of his/her wealth, i.e., the resulting level of wealth at any instant has no relevance for portfolio decisions. Moreover, due to the logarithmic utility, the correlation coefficient, \( \rho \), plays no role in the consumer’s optimal portfolio, only matters the trend and volatility components of the stochastic processes driving the dynamics of the exchange rate and the expected labor income. Finally, it is important to point out that Eq. (18) is cubic; therefore, it has at least one real root.

Notice also that from \( \theta_1 = r^{-1} \), we now have as the solution of (19) to be (see Appendix B)

\[
\phi(v_t) = \theta_2 v^{1_2} + \theta_3 v^{2_2} - \frac{1}{v} \log(v_t) \left( 1 + \frac{2}{\sigma^2_v + 2\ddot{v}} v_t \right) + \frac{1}{v} \left( \sigma^2_v \left( 2\ddot{v} - 1 \right) \right), \tag{21}
\]

where

\[
\lambda_1 = \frac{4r}{(2\ddot{v} - \sigma^2_v) + \sqrt{(2\ddot{v} - \sigma^2_v)^2 + 8r \sigma^2_v}} \tag{22}
\]

and

\[
\lambda_2 = \frac{4r}{(2\ddot{v} - \sigma^2_v) - \sqrt{(2\ddot{v} - \sigma^2_v)^2 + 8r \sigma^2_v}} \tag{23}
\]

Coefficients \( \theta_2 \) and \( \theta_3 \) are to be determined in such a way that \( \phi(v_0) = 0 \) and \( \phi'(v_0) = 0 \) (see Appendix B). The first initial condition assures that economic welfare,

\[
W = I(a_0, v_0, 0) = F(a_0, v_0) = \theta_0 + \theta_1 \log(a_0 v_0),
\]
is independent of the choice of $\phi$. The second initial condition, $\phi'(v_0)=0$, leads to
\[
\frac{\partial I}{\partial v} \bigg|_{v=v_0} = \frac{1}{rv_0} > 0,
\]
which assures that an increase in $v_0$ improves economic welfare. Of course, this second condition also guarantees a unique solution, $\phi$, of (19).

3.6. A viable allocation of portfolio shares

Eq. (18) is cubic with one negative and two positive roots. This can be seen by intersecting the straight line defined by the right-hand side of (18) with the graph defined by the left-hand side of (18). In such a case, there is only one intersection defining a unique steady-state share of wealth set apart for consumption such that $w^* \in (0,1)$.

4. Policy experiments, comparative statics

We are now in a position to derive the first result: a once-and-for-all increase in the rate of devaluation, which results in an increase in the future opportunity cost of purchasing goods, leads to a permanent decrease in the proportion of wealth devoted to future consumption. To see this, we may differentiate (18) to find that
\[
\frac{\partial w^*}{\partial \pi} = -A^{-1} < 0,
\]
where
\[
A = \left[ \frac{r}{(w^*)^2} + \frac{\lambda \gamma^2}{[1 + \gamma(1 - w^*)]^2} + \sigma_P^2 \right].
\]

Another relevant result is the response of the equilibrium share of real monetary balances, $w^*$, to once-and-for-all changes in the intensity parameter, $\lambda$. A once-and-for-all increase in the expected number of devaluations per unit of time causes an increase in the future opportunity cost of purchasing goods. This, in turn, permanently decreases the proportion of wealth set aside for future consumption. Indeed, after differentiating (18), we get
\[
\frac{\partial w^*}{\partial \lambda} = -\frac{\gamma}{A[1 + \gamma(1 - w^*)]} < 0.
\]
A similar effect is obtained for a once-and-for-all change in the mean expected size of a jump:
\[
\frac{\partial w^*}{\partial \gamma} = -\frac{\lambda}{A[1 + \gamma(1 - w^*)]^2} < 0.
\]

5. Welfare implications

We will now assess the effects of exogenous shocks on economic welfare. As usual, the welfare criterion, $W$, of the representative individual is the maximized utility starting from the
initial real wealth, $a_0$, and the initial tax rate on wealth, $v_0$. In virtue of (14), welfare is given by

$$W(p, \lambda, \gamma, \bar{v}; a_0, v_0) = I(a_0, v_0, 0) = F(a_0, v_0) = \frac{1}{r} \left[ 1 + \log(a_0 v_0) + \log(\gamma^{-1} w^*) \right]$$

$$- \frac{1}{r^2} \left[ (\gamma^{-1} + r + \pi - \sigma_p^2) w^* + \frac{1}{2} (w^* \sigma_p)^2 - \bar{v} + \frac{1}{2} \sigma_v^2 - \lambda \log \left( \frac{1+\gamma(1-w^*)}{1+\gamma} \right) \right],$$

(27)

where we have used the fact that $\phi(v_0) = 0$.

5.1. Effects of exchange-rate shocks on welfare

We now compute the impact on welfare of once-and-for-all changes in the mean expected rate of devaluation, the probability of devaluation, and the expected size of a devaluation. First, notice that under the assumption of logarithmic utility, an increase in the stochastic tax coming from devaluation reduces welfare. Indeed, differentiating (27) with respect to $\pi$, we find

$$\frac{\partial W}{\partial \pi} = -\frac{w^*}{r^2} < 0,$$

(28)

Similarly, exogenous shock on the probability of devaluation will produce a reduction in economic welfare. To see this, it is enough to differentiate (27) with respect to $\lambda$

$$\frac{\partial W}{\partial \lambda} = \frac{1}{r^2} \log \left( \frac{1+\gamma(1-w^*)}{1+\gamma} \right) < 0,$$

(29)

Finally, notice that a once-and-for-all increase in the expected size of a devaluation decreases welfare as

$$\frac{\partial W}{\partial \gamma} = -\frac{1}{r^2} \left[ \frac{\lambda w^*}{(1+\gamma)(1+\gamma(1-w^*))} \right] < 0.$$

(30)

5.2. Effects of income shocks on welfare

Let us now compute the impact on welfare of once-and-for-all changes in the mean expected tax rate of transformation from labor income into real assets $\bar{v}$. In this case, we have

$$\frac{\partial W}{\partial \bar{v}} = \frac{1}{r^2} > 0,$$

(31)

Hence, increasing $\bar{v}$ will lead to an increase in economic welfare.

6. Wealth and consumption

We now derive the stochastic process that generates wealth when the optimal rule is applied. After substituting the optimal share $w^*$ into (11), we get

$$da_t = a_t \left[ \left( \frac{\lambda w^*}{1+\gamma(1-w^*)} + (w^* \sigma)^2 + v_t \right) dt - w^* \sigma dz_t + \left( \frac{1+\gamma(1-w^*)}{1+\gamma} - 1 \right) dN_t \right],$$

(32)
where
\[ v_t = v_0 \exp \left\{ \left( \bar{v} - \frac{1}{2} \sigma_v^2 \right) t + \mathcal{E} \sigma v \sqrt{t} \right\}, \] (33)
and \( \mathcal{E} \sim \mathcal{N}(0, 1) \). The density of \( v_t \), given \( v_0 \), satisfies
\[ f_{v_t|v_0}(x|v_0) = \frac{1}{\sqrt{2\pi t} \sigma_v x} \exp \left\{ -\frac{1}{2} \left( \frac{\log(x/v_0) - \left( \bar{v} - \frac{1}{2} \sigma_v^2 \right) t}{\sigma_v \sqrt{t}} \right)^2 \right\}. \] (34)

We also have
\[ E[v_t|v_0] = v_0 e^{\bar{v}t} \] (35)
and
\[ \text{Var}[v_t|v_0] = v_0^2 e^{2\bar{v}t} \left( e^{\bar{v}t} - 1 \right). \] (36)

The solution to the stochastic differential equation in (32), conditional on \( a_0 \), is (see Appendix A, formula (A.3))
\[ a_t = a_0 e^{\delta t}, \] (37)
where
\[ \delta_t = \eta_t + \bar{\zeta}, \quad \eta_t \sim \mathcal{N}[[F(w^*) + v_t]t, G(w^*)t], \] (38)
\[ \bar{\zeta} = H(w^*) N_t, \] (39)
and\(^2\)
\[ N_t \sim \mathcal{P}(\lambda t). \] (40)

The stationary components of the parameters of the above distributions are
\[ F(w^*) = \frac{\lambda_i w^*}{1 + \gamma (1 - w^*)} + \left( \frac{w^* \sigma_p}{2} \right)^2, \quad G(w^*) = (w^* \sigma_p)^2, \] (41)
and
\[ H(w^*) = \log \left( \frac{1 + \gamma (1 - w^*)}{1 + \gamma} \right). \]

Notice also that
\[ E[\delta_t|v_t] = [F(w^*) + v_t + H(w^*)]t \] (42)
and
\[ \text{Var}[\delta_t|v_t] = \left[ G(w^*) + [H(w^*)]^2 \right]t. \] (43)

\(^2\) \( x \sim \mathcal{P}(a) \) denotes a Poisson random variable \( x \) with mean \( a \).
Moreover, it readily follows that
\[ E[\delta_t] = E\{E[\delta_t|v_t]\} = [F(w*) + v_0e^{\gamma t} + H(w*)]\lambda t, \tag{44} \]
and
\[ \text{Var}[\delta_t] = \text{Var}\{E[\delta_t|v_t]\} + E\{\text{Var}[\delta_t|v_t]\} = t^2v_0^2e^{2\gamma t}(e^{2\gamma t} - 1) + [G(w*) + [H(w*)]^2]\lambda t. \tag{45} \]
Even though \( F(w*) \) is always positive and \( H(w*) \) is always negative for all \( 0 < w* < 1 \), the mean of \( \delta_t, E[\delta_t|v_t] \), remains positive. Indeed, since \( x - 1 - \log(x) \geq 0 \) holds for all \( x > 0 \),
\[ \frac{\gamma w*}{1 + \gamma(1 - w*)} - \log\left(\frac{1 + \gamma}{1 + \gamma(1 - w*)}\right) \geq 0, \]
from where the claim about the sign of \( E[\delta_t|v_t] \) follows. Finally, according to (37), Eqs. (44) and (45) determine the mean and variance of the growth rate of real assets.

6.1. Consumption dynamics

In virtue of (6) and (38), the stochastic process for consumption can be written as
\[ c_t^* = \alpha^{-1}w* a_0 e^{\delta_t}. \tag{46} \]
This indicates that, in the absence of contingent-claims markets, the devaluation risk has an effect on wealth through the uncertainty in \( \delta_t \), that is, uncertainty changes the opportunity set faced by the consumer. On the other hand, the devaluation risk also affects the composition of portfolio shares via its effects on \( w* \). Thus, a policy change will be accompanied by both wealth and substitution effects. Notice that from (46), we can compute the probability that, in a given time interval, certain levels of consumption occur. It is also important to note, regarding (46) and (12), that the assumption that the agent’s time-preference rate is equal to the world’s interest rate does not ensure a steady-state level of consumption. However, we do have a steady-state share of wealth set aside for consumption. We may conclude that uncertainty is the clue to rationalize richer consumption dynamics that could not be obtained from deterministic models. Finally, in virtue of (46), Eqs. (44) and (45) determine the mean and variance of the growth rate of consumption.

6.2. Consumption booms

We will analyze now a policy of the form
\[ \pi_t = \begin{cases} \pi_1 & \text{for } 0 \leq t \leq T, \\ \pi_2 & \text{for } t > T, \end{cases} \tag{47} \]
where \( T \) is exogenously determined, and \( \pi_1 < \pi_2 \), as in Calvo (1986). Notice that in our stochastic setting, there is a lack of credibility even if we do not change the four parameters since agents
always assign some probability to the event of currency devaluation. Let us examine the response of consumption to (47). From (46), we may write
\[
\frac{c_{T+\Delta}}{c_T} = \frac{w^*_2}{w^*_1} \exp\left\{ -\left( \delta_T(\pi_1) - \delta_{T+\Delta}(\pi_2) \right) \right\}.
\]

The exponential above tends to 1 as \( \Delta \to 0^+ \) a.s. (almost surely). This means that although the stationary components of the random variable \( \delta_t \) and are different before and after time \( T \), such a difference becomes negligible when \( \Delta \to 0^+ \). Consequently,
\[
\lim_{\Delta \to 0^+} c_{T+\Delta} = c_T \frac{w^*_2}{w^*_1} \quad \text{a.s.} \tag{48}
\]

We also notice that \( w^*_2/w^*_1 < 1 \), together with (48), imply \( c^*_T \lim_{\Delta \to 0^+} c^*_T = c^*_T \) a.s., indicating a jump (boom) in consumption at time \( T \). In other words, if the plan is expected to be temporary, then there is a jump in consumption at \( T \), as we have shown above. Therefore, Calvo’s (1986) deterministic result on the response of consumption to temporary stabilization is locally maintained (around \( T \) a.s.) in our stochastic setting. Notice that the findings are related to those of Calvo and Drazen (1997) with no contingent assets. A similar analysis can be applied to any of the remaining parameters determining the expectations of devaluation, namely, \( \lambda \) and \( \gamma \).

7. Simulation exercise

The following experiment is intended to simulate, via Monte Carlo methods, the response of consumption to permanent changes in the values of the parameters that determine the expectations of devaluation. Table 1 presents two vectors of parameter values, \((\pi_j, \sigma_j^{-1}, \lambda_j, \gamma_j)\), \( j = 1, 2 \), that reproduce the trend and jump of the observed consumption in Mexico between 1989 and 1994.

In Fig. 1, the light solid line represents the simulated trend of consumption without labor income, and the heavy solid line the trend of consumption with labor income. We have taken \( \alpha = 1, r = 0.08, \bar{v} = 0.02, \sigma_v = 0.20, \) and suitable choices of \( T \) and \( a_0 \). The dashed line corresponds to observed consumption. Notice that, with the above parameter values, the stochastic simulation with income labor mimics the order of magnitude of the consumption jump observed

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3 In order to choose a pair of vectors replicating stylized facts, we tried about 800 different feasible combinations of parameter values.

4 For simulation purposes, we have used a standard discrete-time version of (46) with an appropriate unit of time, see, for instance, Ripley (1987) and Press et al. (1992). Here, the critical part of Monte Carlo simulation is the simulation of a Brownian motion combined with a jump process by generating independent random numbers drawn from both the normal and the Poisson distributions. The consumption trend is estimated as an average of simulated paths using (46) repeatedly. Results are based on 10,000 iterations.

5 Data Source: INEGI.
in the first quarter of 1993: a jump of about 60 thousands of millions of pesos of 1993. Without the presence of labor income, the magnitude of the consumption jump is almost preserved but the simulated trend is flatter.

8. Conclusions

The “credibility literature” has by now exhausted a class of deterministic models aimed at explaining consumption dynamics. Most of the existing models ignore uncertainty providing elaborate economic justifications to avoid technical difficulties when risk factors need to be considered. After all, what produces expected temporariness is uncertainty. We have presented a stochastic model of exchange-rate-based stabilization with imperfect credibility. An important feature of our formulation is that there is a lack of credibility even if we do not change the parameters determining the expectations of devaluation.

Our modeling has assumed that agents have expectations of devaluation driven by a mixed diffusion-jump process. In this way, small diffusion movements of the exchange rate, which are always present, are modeled through a Brownian motion, and an extreme and sudden devaluation, which occasionally occurs, is governed by a Poisson process. Mixed diffusion-jump processes provide heavy tails and skewness in the exchange-rate distribution to rationalize richer inflation dynamics that cannot be generated by using only the Brownian motion. This fact is not just a theoretical sophistication but an important issue to be considered in empirical research.

Our proposal, in which stochastic processes drive the expectations of devaluation and future labor income, provides new elements to carry out simulation experiments and
empirical research on some observed regularities that still need to be explained. In particular, our stochastic model was capable of explaining private consumption behavior, in the presence of an uncertain stream of labor income, for the Mexican case of 1989–1994. It is important to point out that in this simulation experiment, the inclusion of risk factors driven by a mixed diffusion-jump process was the clue to deal with some unexplained empirical regularities.

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Appendix A

In this appendix, we state without proof\(^6\) two useful results in the development of this paper:

1) The Itô’s lemma for mixed diffusion-jump processes, which can be stated as follows. Given the homogeneous linear stochastic differential equation

\[ dx_t = x_t(\mu dt + \sigma dz_t + \gamma dq_t), \quad z_t \sim \mathcal{N}(0, t), \quad q_t \sim \mathcal{P}(\lambda t). \]  

(A.1)

and \(g(x_t)\) twice continuously differentiable, then the “stochastic” differential of \(g(x_t)\) is given by

\[ dg(x_t) = \left[ g_x(x_t)\mu x_t + \frac{1}{2} g_{xx}(x_t)x_t^2 \right] dt + g_x(x_t)\sigma x_t dz_t + \left[ g(x_t(1 + \gamma)) - g(x_t) \right] dq_t. \]  

(A.2)

2) The solution to (A.1) is given by

\[ x_t = x_0 \exp \left\{ \left( \mu - \frac{1}{2} \sigma^2 \right) t + \sigma \int_0^t dz_u + \log(1 + \gamma) \int_0^t dq_u \right\}. \]  

(A.3)

It is also worthwhile to keep in mind, when using (A.3), that for \(t \geq 0\) the following properties for \(z_t\) and \(q_t\) hold:

\[ E\left[ \int_0^t dz_u \right] = 0, \quad E\left[ \left( \int_0^t dz_u \right)^2 \right] = \int_0^t du = t, \quad \text{and} \quad E\left[ \int_0^t dq_u \right] = \lambda t. \]

\(^6\) For the proofs, we refer the reader to Gihman and Skorohod (1972, chap. 2).
In this appendix, we solve the nonhomogeneous linear second-order ordinary differential equation stated in (20). Let $\phi = \phi(v)$, and consider the nonhomogeneous Euler–Cauchy type equation

$$v^2 \phi'' + \frac{2\dot{v}}{\sigma^2} v \phi' - \frac{2r}{\sigma^2} \phi = \frac{2}{\sigma^2} \log(v) - \frac{2}{r\sigma^2} v,$$  \hspace{1cm} (B.1)

where $r$ and $\sigma$ are positive constants. In order to transform (B.1) into a differential equation with constant coefficients, we apply Euler’s method by using the change of variable $v = e^t$. Hence, $t = \log(v)$,

$$\frac{\partial \phi}{\partial v} = \frac{1}{v} \frac{\partial \phi}{\partial t},$$  \hspace{1cm} (B.2)

and

$$\frac{\partial^2 \phi}{\partial v^2} = \frac{1}{v^2} \left( \frac{\partial^2 \phi}{\partial t^2} - \frac{\partial \phi}{\partial t} \right).$$  \hspace{1cm} (B.3)

After substituting (B.2) and (B.3) in (B.1), we obtain

$$\frac{\partial^2 \phi}{\partial t^2} + \left( \frac{2\dot{v}}{\sigma^2} - 1 \right) \frac{\partial \phi}{\partial t} - \frac{2r}{\sigma^2} \phi = \frac{2}{\sigma^2} t - \frac{2}{r\sigma^2} e^t.$$  \hspace{1cm} (B.4)

The general solution of this equation is of the form:

$$\phi(t) = \phi_c(t) + \phi_p(t),$$  \hspace{1cm} (B.5)

where $\phi_c$ is the complementary function associated with the homogeneous equation, and $\phi_p$ is a single particular solution of the nonhomogeneous equation. To find $\phi_c$, we need to solve first the following characteristic equation:

$$\lambda^2 + \left( \frac{2\dot{v}}{\sigma^2} - 1 \right) \lambda - \frac{2r}{\sigma^2} = 0.$$ 

Hence, the complementary function is

$$\phi_c(t) = \theta_2 e^{\lambda_1 t} + \theta_3 e^{\lambda_2 t},$$  \hspace{1cm} (B.6)

where the two roots are given by

$$\lambda_1 = \frac{4r}{(2\dot{v} - \sigma^2) + \sqrt{(2\dot{v} - \sigma^2)^2 + 8r\sigma^2}}$$

and

$$\lambda_2 = \frac{4r}{(2\dot{v} - \sigma^2) - \sqrt{(2\dot{v} - \sigma^2)^2 + 8r\sigma^2}}.$$
To find now $\phi_p$, we may use the method of undetermined coefficients. Let us try the guess

$$\phi_p(t) = At + B + Cte', \quad (B.7)$$

so $\phi_p'(t) = A + C(t + 1)e'$ and $\phi_p''(t) = C(t + 2)e'$. After substituting (B.7) in Eq. (B.4), we get

$$\left( \frac{2\bar{v}}{\sigma^2} - \frac{2r}{\sigma^2} \right) Cte' + \left( 1 + \frac{2\bar{v}}{\sigma^2} \right) Ce' - \frac{2r}{\sigma^2} A + \left( \frac{2\bar{v}}{\sigma^2} - 1 \right) A - \frac{2r}{\sigma^2} B = \frac{2}{\sigma^2} t - \frac{2}{r\sigma^2} e'. \tag{B.8}$$

Solving for the coefficients $A$, $B$, and $C$, we obtain

$$A = -\frac{1}{r}, \quad B = \frac{1}{2r^2} \left( \sigma^2 - 2\bar{v} \right), \quad \text{and} \quad C = -\frac{2}{r(\sigma^2 + 2\bar{v})},$$

and find that for a particular solution we must have $\bar{v} = r$. Therefore,

$$\phi_p(t) = \frac{1}{\bar{v}} t - \frac{1}{\bar{v}} + \frac{\sigma^2}{2\bar{v}^2} - \frac{2}{\bar{v}(\sigma^2 + 2\bar{v})} te'. \tag{B.8}$$

Substituting (B.6) and (B.8) into (B.5) leads to

$$\phi(t) = \theta_2 e^{\bar{v}t} + \theta_3 e^{\bar{v}t} - \frac{1}{\bar{v}} t - \frac{1}{\bar{v}} + \frac{\sigma^2}{2\bar{v}^2} - \frac{2}{\bar{v}(\sigma^2 + 2\bar{v})} te'. $$

Since $v = e'$, the general solution of (B.1), in terms of $v$, is given by

$$\phi(v_t) = \theta_2 v_t^{\bar{v}2} + \theta_3 v_t^{\bar{v}2} - \frac{1}{\bar{v}} \log(v_t) \left( 1 + \frac{2}{(\sigma^2 + 2\bar{v})} v_t \right) + \frac{1}{\bar{v}} \left( \sigma^2 - 2\bar{v} \right). \tag{B.9}$$

The values of $\theta_2$ and $\theta_3$ satisfying the initial conditions $\phi(v_0) = \phi'(v_0) = 0$ are

$$\theta_2 = \frac{v_0^{-\bar{v}2}}{(\lambda_1 - \lambda_2)v} \left[ \lambda_2 \left( \log(v_0) + 1 - \frac{\sigma^2}{2\bar{v}} \right) - \frac{2v_0}{2\bar{v} + \sigma^2} (1 + \log(v_0)(1 - \lambda_2)) + 1 \right]$$

and

$$\theta_3 = \frac{v_0^{-\bar{v}2}}{(\lambda_1 - \lambda_2)v} \left[ - \lambda_1 \left( \log(v_0) + 1 - \frac{\sigma^2}{2\bar{v}} \right) + \frac{2v_0}{2\bar{v} + \sigma^2} (1 + \log(v_0)(1 - \lambda_1)) + 1 \right].$$

References